

Question 1

1) Cobb-Douglas specification에 대한 log의 가정식:

$$\ln Y_i = \beta_0 + \beta_L \ln L_i + \beta_K \ln K_i + u_i \quad (1)$$

(1) equation을 $\ln L_i$ 의 편미분

$$\frac{\partial \ln Y_i}{\partial \ln L_i} = \beta_L$$

β_L , β_{KL} 100 unit Elasticity를 의미하며, β_L 은 β_{KL} 의 1/2이다.

모든 data / β_L 은 β_{KL} 의 1/2이다

β_L 은 β_{KL} 의 1/2이다

MLE보다 MLE의 효율성이 높으므로, $\beta_L^d > \beta_L^g$ ($d = \text{MLE}, g = \text{OLS}$)

이러한 경우 MLE의 효율성이 높으므로, $\beta_L^d < \beta_L^g$

가정식에서 MLE의 효율성이 높으므로, $\beta_0^d < \beta_0^g$ 의

관계를 가진 것이다

이러한 경우

2) 가정에 따르면 MLE의 효율성이 높으므로 consistency 또한

보지만, β_L 은 β_{KL} 의 1/2이므로 MLE의 효율성이 높으므로

High-productive firm, less workers를 고용하는 것으로 보일 수 있다

β_L 의 편미분을 보라

2014. OLS에 비해 MLE의 효율성이 높으므로 MLE의

측정에 measurement error가 생길 때 attenuation bias를

개선 β_L 은 inconsistency 보임

이러한 High productive firm의 β_L 이 under-estimate.

more productive

$$\frac{1}{K} = \frac{1}{L} \cdot \frac{L}{K}$$

3. Endogeneity로 인해 β_0 이 under-estimated 되므로 3가지 방법이

있을 수 있다.

1) 2단계 MLE에 대한 비선형 2단계 MLE의 β_L 을 구한다

Output 생략 후 β_L 을 β_L^H 를 구한 후 기존의 β_L 과 비교하면

$\beta_L^H > \beta_L$ 이 성립할 것이다. 이를 통해 전체 평균으로

β_L 을 추정하면 High Productive firm들의 β_L 이 under-estimate

되므로 알 수 있다.

2) β_L 을 대입하면, 새로운 변수를 추가한다

$$Y_i = \beta_0 + \beta_L L_i + \beta_S S_i + \beta_K K_i + u_i$$

$\rightarrow Y_i = \beta_0 + \beta_L L_i + \beta_S S_i + \beta_K K_i + u_i$

3) IV를 이용하여 β_L 의 편미분을 OLS로 구한 β_L 과

비교하면 $\beta_L^{IV} > \beta_L^{OLS}$ 라면 β_L 이 under-estimate

4. IV의 편미분은 $E(Z \cdot u) = 0$ (Z = IV)

① Validity: $E(Z \cdot u) = 0$

② Relevance: $E(Z \cdot L) \neq 0$

이들 편미분은 IV를 보라, β_L , β_K , β_S 의 편미분 등이 있다

5. 2SLS를 통한 Endogeneity를 해결하는 방법

$$\ln Y_i = \beta_0 + \beta_L \ln L_i + \beta_K \ln K_i + u_i \quad (\text{Equation 1})$$

$$1\text{-step } \ln L_i = \beta_0 + \beta_L \ln L_i + \beta_K \ln K_i + u_i$$

$$\ln L_i = \beta_0 + \beta_L \ln L_i + \beta_K \ln K_i + u_i$$

$$2\text{-step } \ln Y_i = \beta_0 + \beta_L \ln L_i + \beta_K \ln K_i + v_i$$

이러한 2SLS를 사용하면, Z_i 가 4000, 5000, 6000, 7000, 8000, 9000, 10000, 11000, 12000, 13000, 14000, 15000, 16000, 17000, 18000, 19000, 20000, 21000, 22000, 23000, 24000, 25000, 26000, 27000, 28000, 29000, 30000, 31000, 32000, 33000, 34000, 35000, 36000, 37000, 38000, 39000, 40000, 41000, 42000, 43000, 44000, 45000, 46000, 47000, 48000, 49000, 50000, 51000, 52000, 53000, 54000, 55000, 56000, 57000, 58000, 59000, 60000, 61000, 62000, 63000, 64000, 65000, 66000, 67000, 68000, 69000, 70000, 71000, 72000, 73000, 74000, 75000, 76000, 77000, 78000, 79000, 80000, 81000, 82000, 83000, 84000, 85000, 86000, 87000, 88000, 89000, 90000, 91000, 92000, 93000, 94000, 95000, 96000, 97000, 98000, 99000, 100000, 101000, 102000, 103000, 104000, 105000, 106000, 107000, 108000, 109000, 110000, 111000, 112000, 113000, 114000, 115000, 116000, 117000, 118000, 119000, 120000, 121000, 122000, 123000, 124000, 125000, 126000, 127000, 128000, 129000, 130000, 131000, 132000, 133000, 134000, 135000, 136000, 137000, 138000, 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639000, 640000, 641000, 642000, 643000, 644000, 645000, 646000, 647000, 648000, 649000, 650000, 651000, 652000, 653000, 654000, 655000, 656000, 657000, 658000, 659000, 660000, 661000, 662000, 663000, 664000, 665000, 666000, 667000, 668000, 669000, 670000, 671000, 672000, 673000, 674000, 675000, 676000, 677000, 678000, 679000, 680000, 681000, 682000, 683000, 684000, 685000, 686000, 687000, 688000, 689000, 690000, 691000, 692000, 693000, 694000, 695000, 696000, 697000, 698000, 699000, 700000, 701000, 702000, 703000, 704000, 705000, 706000, 707000, 708000, 709000, 710000, 711000, 712000, 713000, 714000, 715000, 716000, 717000, 718000, 719000, 720000, 721000, 722000, 723000, 724000, 725000, 726000, 727000, 728000, 729000, 730000, 731000, 732000, 733000, 734000, 735000, 736000, 737000, 738000, 739000, 740000, 741000, 742000, 743000, 744000, 745000, 746000, 747000, 748000, 749000, 750000, 751000, 752000, 753000, 754000, 755000, 756000, 757000, 758000, 759000, 760000, 761000, 762000, 763000, 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1012000, 1013000, 1014000, 1015000, 1016000, 1017000, 1018000, 1019000, 1020000, 1021000, 1022000, 1023000, 1024000, 1025000, 1026000, 1027000, 1028000, 1029000, 1030000, 1031000, 1032000, 1033000, 1034000, 1035000, 1036000, 1037000, 1038000, 1039000, 1040000, 1041000, 1042000, 1043000, 1044000, 1045000, 1046000, 1047000, 1048000, 1049000, 1050000, 1051000, 1052000, 1053000, 1054000, 1055000, 1056000, 1057000, 1058000, 1059000, 1060000, 1061000, 1062000, 1063000, 1064000, 1065000, 1066000, 1067000, 1068000, 1069000, 1070000, 1071000, 1072000, 1073000, 1074000, 1075000, 1076000, 1077000, 1078000, 1079000, 1080000, 1081000, 1082000, 1083000, 1084000, 1085000, 1086000, 1087000, 1088000, 1089000, 1090000, 1091000, 1092000, 1093000, 1094000, 1095000, 1096000, 1097000, 1098000, 1099000, 1100000, 1101000, 1102000, 1103000, 1104000, 1105000, 1106000, 1107000, 1108000, 1109000, 1110000, 1111000, 1112000, 1113000, 1114000, 1115000, 1116000, 1117000, 1118000, 1119000, 1120000, 1121000, 1122000, 1123000, 1124000, 1125000, 1126000, 1127000, 1128000, 1129000, 1130000, 1131000, 1132000, 1133000, 1134000, 1135000, 1136000, 1137000, 1138000, 1139000, 1140000, 1141000, 1142000, 1143000, 1144000, 1145000, 1146000, 1147000, 1148000, 1149000, 1150000, 1151000, 1152000, 1153000, 1154000, 1155000, 1156000, 1157000, 1158000, 1159000, 1160000, 1161000, 1162000, 1163000, 1164000, 1165000, 1166000, 1167000, 1168000, 1169000, 1170000, 1171000, 1172000, 1173000, 1174000, 1175000, 1176000, 1177000, 1178000, 1179000, 1180000, 1181000, 1182000, 1183000, 1184000, 1185000, 1186000, 1187000, 1188000, 1189000, 1190000, 1191000, 1192000, 1193000, 11

2SLS를 사용할 때 $\hat{\beta}_{2SLS}$, $\hat{\beta}_{OLS}$, $\hat{\beta}_{IV}$ 를 구할 수 있으며,
 $AM3$ 부분 β_{2SLS} 값이 β_{OLS} 의 값보다 크다면 LM^{*} 는
 긍정적 상관관계로, β_{IV} $\hat{\beta}_{OLS}$ 를 줄여줄 것을 의미한다.

6. 회사가 임금차이가 크지 않다면 가정에 Measurement error가

β_{OLS} Under-estimate 된다는 것을 설명하기 어렵다.

즉, β_{OLS} 가 β_{IV} 의 값을 가리키지 않다면 Measurement error가
 양의 것이다.

이런 상황에서 $AM3$ 부분이 β_{OLS} Under-estimate 되었다는 것을
 판정하기 위해 LM^{*} 값이 0에 가까울수록 양의 상관관계가

$$\beta_{OLS} = \frac{\sigma_{LM^{*}}}{\sigma_{LM^{*}} + \sigma_{\epsilon}} \quad \text{이므로 } LM^{*} \text{가 작을수록 크게 줄어듦을 의미,}$$

β_{OLS} 가 작아지기 때문이다. 예를 들어, 실제 임박 시간보다 기록된
 업무시간이 더 길 경우 이런 오류가 발생한다.



Company의 Data Scientist는 노동력을 구한 후
 모든 직원을 한 직장으로 분류한 것이 아닌
 사업장인 위와 같. 예를 들어, (1) 비효율
 이 많은 것이 β_{OLS} 를 감소시키고, β_{IV} 를 증가

시킴을 의미한다.

즉, $AM3$ 양의 상관관계는 OLS 보다 Comparative advantage를
 가지고 있는 회사이므로 해당부의 직원을 한 직장으로 분류한 노동력을
 이용한다면, β_{OLS} 는 양의 상관관계가 아니다.

즉, $LM^{*} = \beta_{OLS} + \beta_{IV} + \beta_{IV} + \beta_{IV} + \beta_{IV} + \beta_{IV}$ 식이기

때문에 이보다, OLS 의 β_{OLS} 가 IV 의 β_{IV} 보다 크면 LM^{*} 가 양의

상관관계가 된다. LM^{*} Management, Communication... 등이 negative한
 영향을 받는다. 결국 LM^{*} 는 양의 상관관계가 된다.
 β_{OLS} 이 양의 상관관계가 된다.