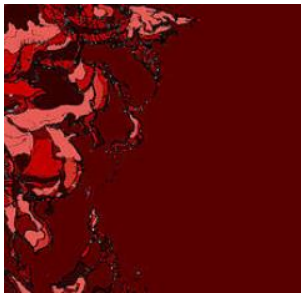
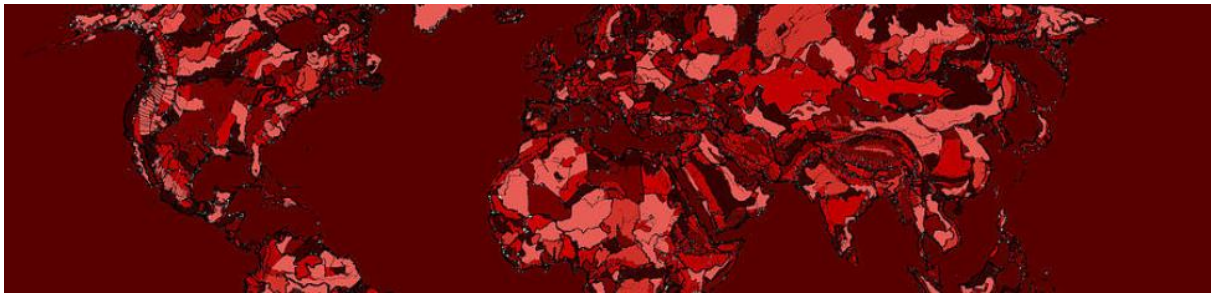
STA501: Data-based Decision Making

Lecture 2: Consumer Choices

Swiss Institute of
Artificial Intelligence





Elasticity – One's proportional response to the other at one point

- Elasticity is a building block of proportional response in science

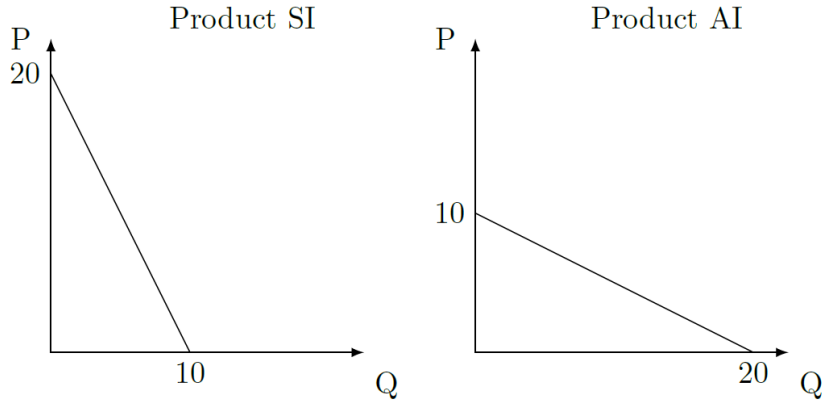
Elasticity

Definition of elasticity

- The changes in quantity y due to changes in quantity x

$$
\begin{aligned}
e_{xy} &= \frac{\triangle y/y}{\triangle x/x} = \frac{\triangle y}{\triangle x} \times \frac{x}{y} \\
&= \text{Proportional change} \times \text{(X, Y) inverse ratio} \\
&= \text{Slope at point} \times \text{Slope from origin} \\
&= \frac{d \ln y}{d \ln x} \quad \text{(at limit)}
\end{aligned}
$$

Examples in terms of price elasticity of demand



- As a seller, for which product would you raise price?

Extensions – N goods case, T periods case

N goods case

- Imagine NxN matrix of price elasticity of multiple goods demand
  - In real world, a lot of consumer goods are consumed in a bundle with varying ratio, and many can be substituted by another (near impossible job to do…)

T periods case

- Is an elasticity (of anything) constant?
  - For some goods to prices, yes, but for many other products, there are competitors in the market and people's preference changes over time

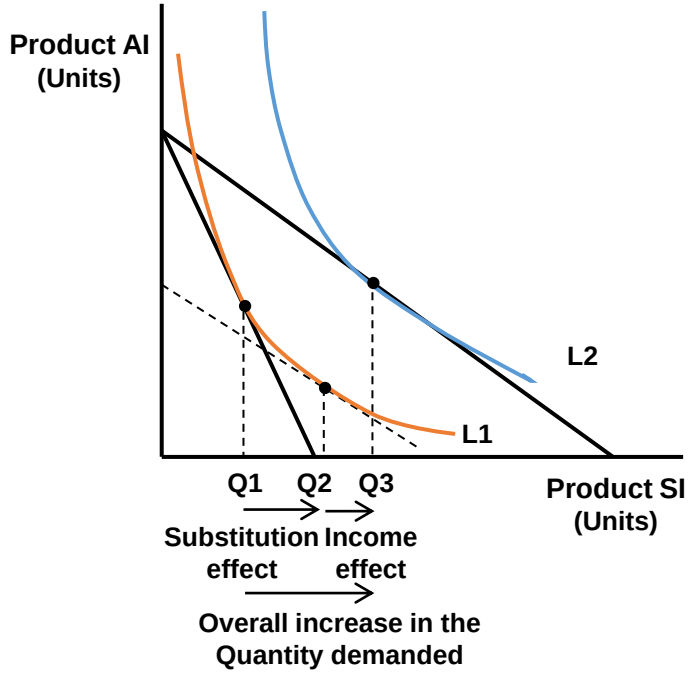
- The inherent complication in measuring cross-product elasticity in different times makes the concept of "elasticity" hardly a real world applicable case for analysis, unless you have a framework of thinking

Demand decomposed in relative price

- Substitution effect vs. Income effect

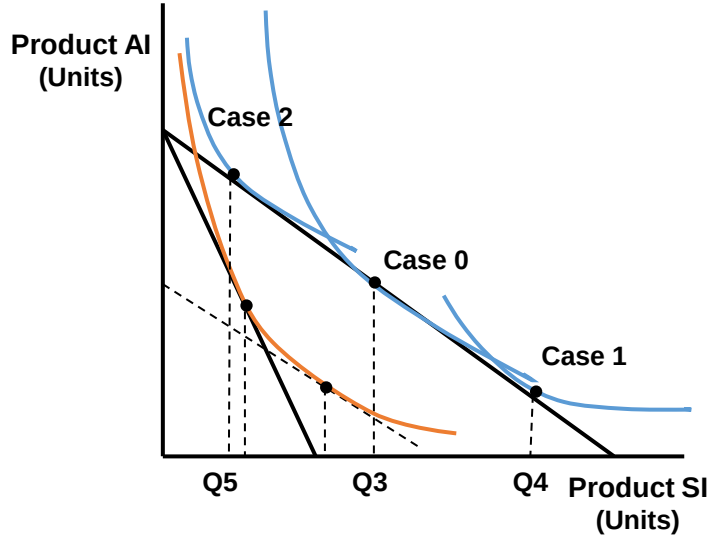
Substitution vs. Income effects

Slutsky decomposition



- Lower price to more demand for goods can be decomposed into two parts
  - Substitution from AI to SI
  - (Relatively more) Income for both SI and AI

Varing income effects for goods

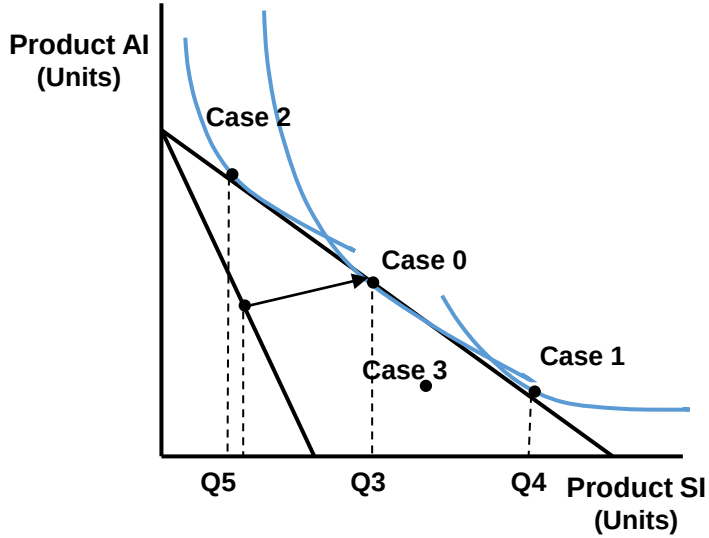


- Income effects can be varying depending on MRS
  - Q4 means extreme price elasticity of demand for SI
  - Q5 means the income effect works in opposite direction (known as “Giffen good”)
- As a seller, your pricing strategy should always take into account the elasticity (thus income effect)
- But only ex-post consumption is visible, not the demand
- Then, how to apply the concept to real world?

Revealed preference – a realization of hidden demand

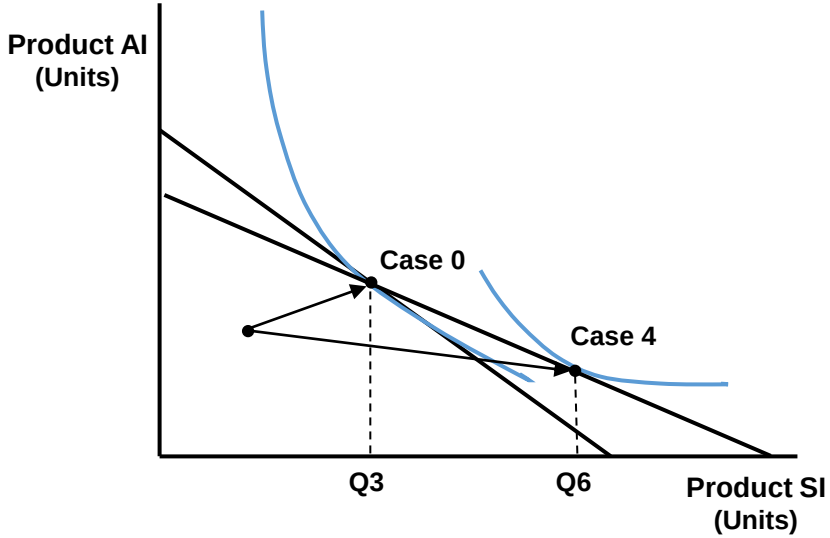
- When theory meets limit, data tells what it really is

Revealed preference – what has not been chosen



- When SI became cheaper, the consumer chose the bundle at Case 0, not Case 1, 2, 3
  - Assuming the consumer is “rational”, at least aggregate consumers are “rational”, then, Case 0 must be the “rationally preferred” choice
  - Even if rationality is missing, still the chosen bundle is the “preferred”, which is available as “data”
  - The “Preference” is “Revealed” in “data”

What if other options were available?

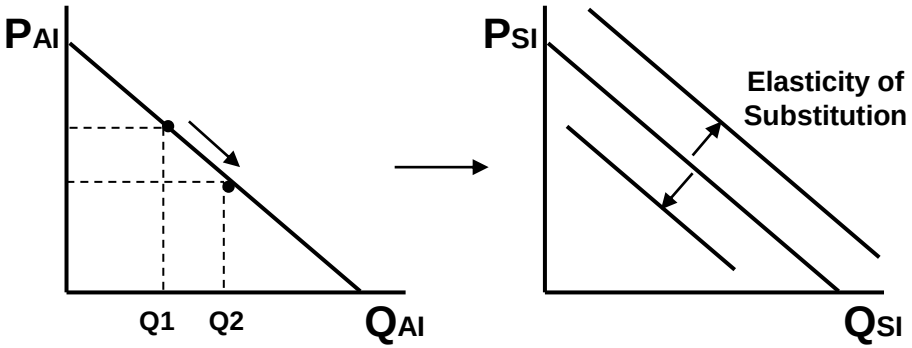


- Assume an identical twin city had different transportation cost and price for SI and AI are different
- If Case 4 was chosen, instead of Case 0, by the revealed preference, the data tells us
  - Income effect on SI’s demand
  - Elasticity of SI’s demand
  - Preference of SI over AI
- Still, it is a demand at one or two prices, not for all prices

Substitutes and complements

- Relations between two (or more) products

Definition of substitutes and complements



- When AI's price drops and consumption increases,
  - If SI is demanded more, it is complements
  - If SI is demanded less, it is substitutes
- Unless one good is consumed in isolation, all goods' demand must be measured in basket
  - Estimating demand for all prices for a single good is nearly an impossible task
  - Instead, take below two (or three) steps
    - Separate other goods' influence in one good's demand by proper market definition
    - Measure elasticity in small price difference
    - (Extrapolate the elasticity to create a demand curve)

Market definition – Spirits & Beer

- In 2005, Korea Fair Trade Committee (KFTC) approved a merger of nation's leading brewery (Hite) and dominant local spirits provider (Jinro) that there are noticeable synergy in sales channel, but no supply or demand of two products are complements
  - Are they really substitutes?
  - What if both are substitutes for some people and complements for the others?
  - How do they measure substitutability?
    - Small but Significant Non-transitory Increase in Price (SSNIP) or Critical Loss Analysis (CLA)

Source: https://www.intechopen.com/chapters/50672

Substitutes and complements (cont'd)

- Estimations and interpretations based on poll (Why poll data….)

Price of spirits

Elasticity of substitution due to changes in local spirits price

		Consumption	% difference	Elasticity
Before price increase	Local spirit	13,784		
	Beer	8,145		
	Foreign spirits	452		
5% price increase	Local spirit	13,010	-5.60	-1.12
	Beer	8,478	4.10	0.82
	Foreign spirits	462	2.20	0.44
10% price increase	Local spirit	12,319	-10.60	-1.06
	Beer	8,939	9.70	0.97
	Foreign spirits	468	3.50	0.35

Note: Per 360ml, 500ml, 500ml bottles for local spirits, beer, and foreign spirits

Source: Gallup Korea

- Demand for local spirit is price elastic (as >1), but the scale is diminishing
- Demand is certainly (and mostly) shifted to beer, and the magnitude is increasing (0.82 -> 0.97)
- Can you tell if local spirit is a Giffen good?
  – Will it be, if we have upto 100% price increase?
- Are they substitutes?

Market definition – Spirits & Beer

Elasticity of substitution due to changes in beer price

		Consumption	% difference	Elasticity
Before price increase	Local spirit	9,058		
	Beer	12,625		
	Foreign spirits	405		
5% price increase	Local spirit	9,596	5.90	1.18
	Beer	10,960	-13.20	-2.64
	Foreign spirits	417	2.80	0.56
10% price increase	Local spirit	10,062	11.10	1.11
	Beer	9,840	-22.10	-2.21
	Foreign spirits	468	4.70	0.47

Note: Per 360ml, 500ml, 500ml bottles for local spirits, beer, and foreign spirits

Source: Gallup Korea

- Demand for beer is way more sensitive than for local spirit (as >2), but the scale is diminishing
- Demand is only partly shifted to beer, and the magnitude is decreasing (1.18 -> 1.11)
- Is the lower demand due to income effect or preference?
  – How to measure income effect? Any data needed?
- How relevant is foreign spirits in this analysis?

Multi-factor model extension

- Logged regression is to measure elasticity

Regression and elasticity

- Below Cobb-Douglas function is a famous non-linear combination of two inputs for an output

$$Q = f(L, K) = AL^{\alpha}K^{\beta}$$
$$\log Q = \log A + \alpha \log L + \beta \log K$$
$$y = a + \alpha x_1 + \beta x_2$$

- It can be translated to a simple regression (if properly logged to de-power the variables

$$e_{LQ} = \frac{d \ln Q}{d \ln L} = \frac{dy}{dx_1} = \alpha$$
$$e_{KQ} = \frac{d \ln Q}{d \ln K} = \frac{dy}{dx_2} = \beta$$

- The coefficients are elasticity of inputs to output at currently used point (Revealed value in data)
- Can be extended to N variables case and T periods case
- If sum of the powers are equal to 1, in economics it is called homogenous of degree 1, meaning output is the same amount as all inputs (i.e. exactly proportional)

Market definition – Spirits & Beer – What if?

- Was the single variable regression (the limit of SSNIP test) enough to test substitutability?
- Any possibility to test it, based on revealed preference?
  - Hasn't the historical price of two products changed dis-proportionately at all?
  - Are they jointly consumed, and if so, how to find the relevant data set and how to put it into a regression?
  - What does the joint consumption mean, and if the proportion of joint consumption is 30%, what can you say about substitutability?
  - Any reason to run separate regression for joint and dis-joint consumption?
- A candidate regression should include
  - Q_beer, Q_local spirit
  - P_beer, P_local spirit
  - Joint consumption index (dummy variable?)
  - Where to consume
  - Other missing factors…

Multi-factor model extension (cont'd)

- An example of multi-factor model in log form

An example non-linear function: $C = Y \times P_L \times P_{K1} \times P_{K2}$

Question 2. In the article "Returns to Scale in Electricity Supply", the following cost function was estimated by Ordinary Least Squares (OLS) using data on different firms:

$$\log(\text{Cost}) = -\underset{(1.77)}{3.53} + \underset{(0.0175)}{0.720}\log(\text{Output}) + \underset{(0.291)}{0.436}\log(\text{Price of Labor}) + \underset{(0.339)}{0.220}\log(\text{Cost of Capital}) + \underset{(0.100)}{0.427}\log(\text{Price of Fuel}) + \hat{\epsilon}$$

where $R^2 = 0.93$. Standard errors in parantheses. Number of observations 145.

Estimated Covariance matrix of the estimated coefficients.

Constant	3.148				
Log(Output)	-0.437E-02	0.305E-03			
Log(PriceofLabor)	-0.141	-0.455E-03	0.0847		
Log(CostofCapital)	0.591	0.323E-03	0.0237	0.115	
Log(PriceofFuel)	0.715E-02	0.315E-03	-0.0109	-0.663E-02	0.0101

1. Test the hypothesis that the electricity industry displays constant returns to scale against the alternative that it displays increasing returns at the 5% significance level.
2. We would expect the cost function to be homogeneous of degree one in prices. What does this imply for the coefficients of the cost function? Test the hypothesis that the cost function is homogeneous of degree one (i.e. produce just as much input invested) against the alternative that it is not at the 5% significance level.

Multi-factor model extension (cont'd)

- An example of multi-factor model in log form

Solution manual + application to market definition case

1. To test at 5% significance level $H_0 : \beta_2 = 1$ vs. $H_1 : \beta_2 < 1$, we define the one-sided test procedure:
   $\tau = (\hat{\beta}_2 - 1)/SE(\hat{\beta}_2) < t^*_{5\%}(140) = -1.655$
   Do not reject H_0 iff $\tau \geq t^*$.
   In this case, $\tau = (0.720 - 1)/0.0175 = -16 < -1.655$, thus we reject H_0.

2. To test the price homogeneity of degree one $H_0 : \beta_3 + \beta_4 + \beta_5 = 1$ vs. $H_1 : \beta_3 + \beta_4 + \beta_5 \neq 1$ at 5% significance level, we follow the decision rule:
   Do not reject H_0 iff $|\tau| = |(\hat{\beta}_3 + \hat{\beta}_4 + \hat{\beta}_5 - 1)/SE(\hat{\beta}_3 + \hat{\beta}_4 + \hat{\beta}_5 - 1)| < t^*_{2.5\%}(14) = 1.977$;
   Reject H_0 otherwise.
   Where $SE(\hat{\beta}_3 + \hat{\beta}_4 + \hat{\beta}_5 - 1) = \sqrt{\hat{V}_3 + \hat{V}_4 + \hat{V}_5 + 2(\hat{V}_{34} + \hat{V}_{35} + \hat{V}_{45})}$. In this case, $\tau = (0.436 - 0.220 + 0.427 - 1)/\sqrt{0.0847 + 0.115 + 0.0101 + 2(0.0237 + 0.0109 + 0.00663)} = -0.7575$, thus, H_0 is not rejected.

- Assuming the same logic to the aforementioned market definition case,
  - To measure substitutability of local spirit to beer, one needs to separate income effect and substitution effect, which can be done with multi-factor model as the presence of the other good's demand and price controls for income effect
  - What about snacks go with beer and/or local spirit? Can the variable(s) help us to determine true substitutability of local spirt to beer? What about separating income effect?
- How would you test the claim that price of any alcohol would not affect the total aggregate consumption of alcohol? Any insight from homogeneity of degree 1?