

### Question 1-a

[Answer] As we have prices on the linear regression, the coefficients will be the price elasticity on GDP. The next 2 years means 8 quarters. If we replace  $t$  with  $t + 8$ , the the linear regression will be like this:

$$\hat{Y} = 1.0 - 0.055Q_{t+8} - 0.026Q_{t+7} - 0.031Q_{t+6} - 0.109Q_{t+5} - 0.128Q_{t+4} + 0.038Q_{t+3} + 0.025Q_{t+2} - 0.019Q_{t+1} + 0.067Q_t \quad (1)$$

The coefficient of  $Q_t$  means the effect of oil price jump in this year based on last year's peak. So what we want to know is the following 8 year's coefficients multiplied by 25%.

$$[-0.019 \times 25, 0.025 \times 25, 0.038 \times 25, -0.128 \times 25, -0.109 \times 25, -0.031 \times 25, -0.026 \times 25, -0.055 \times 25] \quad (2)$$

### Question 1-b

[Answer] From the research of 8 quarters, we see that except two quarters, the increase in oil price will have negative effect on GDP. We can intuitively suggest that the increase in oil price hinders gross domestic product.

### Question 1-c

[Answer] From 1955 to 2020, we have  $66 \times 4 (=264)$  data points consisting of (oil price percentage difference, GDP ratio). If we apply t test, the degree of freedom is  $254 = 264 - 10$  (terms in linear regression). If we think about 95% confidence interval, then we use  $t_{(2.5\%)} = 0.675$ . If a coefficient is  $\beta_k$ , its confidence interval is  $25\hat{\beta}_k \pm t_{0.25}\sigma_{\hat{\beta}_k} = 25\hat{\beta}_k \pm 0.675\sigma_{\hat{\beta}_k}$ . For example, the first quarter's effect is:

$$[-0.019 \times 25 \pm 0.675 \times 0.039] = [-0.019 \times 25 - 0.675 \times 0.039, -0.019 \times 25 + 0.675 \times 0.039] = [-0.449, -0.501] \quad (3)$$

### Question 1-d

[Answer] Assuming that k year's effect is  $\beta$ , then sum of all effect is:

$$25 \times (\beta_1 + \dots + \beta_8) = 25 \times (-0.019 + 0.025 + \dots - 0.026 - 0.055) = -7.625 \quad (4)$$

### Question 1-e

[Answer] F statistics is like:

$$F = \frac{(TSS - RSS)/(p - 1)}{RSS/(n - p)} = F(9, 254) \quad (5)$$

(Sorry. This time I do not know how to find a value from f statistics table. So I copy from woopil's answer) Since 5% critical value of  $F(9, 254)$  is 2.0519, we can reject the null hypothesis where all  $\beta$ s are zero and conclude the coefficients are significantly different from zero. Robust F statistics is required because omitted variable such as interest rate is included in error term and interest rate is serially correlated: Interest reate tends to be low in recessions and high in expansions. Therefore, error term also becomes serially correlated and we introduced robust F-statistics to solve this problem.

### Question 2-a

[Answer] There are substitution effect and income effect at the same time. In A's equation, there is no factor for demand of beer, which is necessary for separating the substition effect from its income effect. Mis-specification in regression.

### Question 2-b

[Answer] Due to the use of logarithm, it is not necessary to scale the demand of spirit by price of spirit.

**Question 2-c**

[Answer]  $\beta_2$ ,  $\beta_3$ ,  $\alpha_2$ , and  $\alpha_3$  are the price elasticity on demand of spirits. If we think about Cobb-Douglas equation to its logarithmic form, we can realize the coefficients can be elasticities. So  $\beta_2$  is the price elasticity on demand of spirits, and  $\beta_3$  is the price elasticity on demand of spirits. However,  $\alpha_2$  and  $\alpha_3$  are meaningless as linear regression is wrong. As linear regression in B is wrong,  $R^2$  in A and B are different.

**Question 2-d**

[Answer] I think that research C is right. The linear regression of A can show elasticity but it includes substitution effect and income effect at the same time. In order to separate income effect, we have to know the relationship between 'demand of spirits' and 'demand of beer'. So we have to have that term in linear equation.

**Question 2-e**

[Answer] For cocktail, we can introduce a new index  $I$  in linear equation like:

$$I = 1(\text{for cocktail}) \text{ or } 0(\text{otherwise}) \quad (6)$$

Then we have the following linear regression:

$$\log(\text{demand of spirits}) = \beta_1 + \beta_2 \log(\text{price of beer}) + \beta_3 \log(\text{price of spirit}) + \beta_4 \log(\text{demand of beer}) + \beta_5 I + \beta_6 I \log(\text{demand of beer}) \quad (7)$$

Now  $\beta_4$  can capture the substitution effect without cocktail consumption and  $\beta_4 + \beta_6$  can capture the substitution effect with cocktail.