

$$1. Y_t = \beta_0 + \beta_1 O_t + \beta_2 O_{t-1} + \dots + \beta_9 O_{t-8} + \epsilon_t$$

(a) The coefficient of O_{t-k} is the effect on Y_t of a unit change in O_{t-k} and this is equivalent to the effect on Y_{t+k} of a unit change in O_t . In other words, the coefficient on O_{t-k} is the effect of a unit change in O_t on Y after k quarters.

Therefore, with 25% oil prices jump, predicted effect on output growth for k th quarter is $25\hat{\beta}_k$ ($k=0,1,2,\dots,8$ since 2 years = 8 quarters) percentage points. For example, for 3rd quarter predicted effect on output growth is $(-0.109) \times 25 = -2.725$ percentage point

(b) The researcher thinks jump of oil prices above their values in the recent past affect the output growth up to after 2 years (8 quarters). Beyond 2 years, it has no effect on the output growth

(c) The 95% confidence interval for k th quarter with 25% oil prices jump is,

$$25 \hat{\beta}_k \pm t_{0.25}(66-10) \text{ s.e.}(25 \hat{\beta}_k) \quad (\because n=66)$$
$$= 25 (\hat{\beta}_k \pm 2.003 \text{ s.e.}(\hat{\beta}_k)) \quad (k=0, 1, 2, \dots, 9)$$

Since we know $\text{s.e.}(\hat{\beta}_k)$ from the problem, we can obtain the 95% confidence interval

For example, the 95% confidence interval for 3th quarter with 25% oil prices jump is,

$$25 (-0.109 \pm 2.003 \times 0.042) = [-4.8285, -0.6485]$$

(d) The predicted cumulative change in GDP growth over 8 quarters is $25 \times (\beta_1 + \beta_2 + \dots + \beta_9)$

$$25 \times (\beta_1 + \dots + \beta_9) = 25 \times (-0.055 - 0.026 + \dots + 0.067)$$
$$= -5.95\%$$

(e) $F = \frac{(RSS - URSS)/df}{URSS/(n-k)} \sim F(9, 66-10) = F(9, 56)$

Since the 5% critical value of $F(9, 56)$ is 2.059, we can reject the null hypothesis: $\beta_1 = \beta_2 = \dots = \beta_9 = 0$ and conclude the coefficients are significantly different from zero. Robust F-statistic is required because

the omitted variable such as interest rate is included in error term ϵ_t and interest rate is serially correlated. Interest rate tends to be low in recessions and high in expansions.

Therefore, error term also becomes serially correlated and we introduced robust F-statistic to solve this problem

2. (a) A's claim is incorrect since the demand of beer is not included as explanatory variable in the regression such that substitution effect can not be separated from income effect

(b) Since the logarithmic term itself can do scaling, additional scaling is not required. Moreover, if we include price of spirits as a denominator of a response, the regression becomes meaningless. Pure effect of price of beer or price of spirits on $\frac{\text{demand of spirits}}{\text{price of spirits}}$ has no meaning.

(c) β_2 means the effect of price of beer on demand of spirits,
 β_3 means the effect of price of spirits on demand of spirits
which is price elasticity of demand in spirits

α_2 means the effect of price of beer on demand of spirits
per price of spirits, α_3 means the effect of price of spirits
on demand of spirits per price of spirits

R^2 will be different because the dependent variable
is different from A & B and we cannot say

R^2 of B is better than that of A. We don't know it

(d) C's claim is correct. Since including both demand
of beer and price of beer/spirits can differentiate
substitution effect from income effect, we need
those three terms. If we omit the demand of beer,
substitution effect will be mixed with income effect
in the coefficient of $\log(\text{Price of Beer})$ and that of $\log(\text{Price of Spirits})$

(e) We can define the joint consumption index I .

$$I = \begin{cases} 1 & \text{if mixture of beer and spirit is consumed} \\ 0 & \text{otherwise} \end{cases}$$

We need the amount of mixture of beer and spirit
and assign 1 if that amount is bigger than 0 and 0
otherwise. To reflect it in the regression
we should not apply logarithmic term to I

Since $\log I$ is not defined when $I=1$ or 0

Rather, we will leave it without logarithmic term

Therefore, the regression form would be,

$$\begin{aligned} \log(\text{demand of spirits}) = & \beta_1 + \beta_2 \log(\text{Price of beer}) + \beta_3 (\text{Price of spirits}) \\ & + \beta_4 \log(\text{demand of beer}) + \beta_5 I \\ & + \beta_6 I \cdot \log(\text{demand of beer}) \end{aligned}$$

Now β_4 can purely capture the substitution effect without joint consumption and

$\beta_4 + \beta_6$ can show the substitution effect with joint consumption