Question 1. Increases in oil prices have been blamed for several recessions in developed countries. Let GDP_t denote the value of quarterly gross domestic product in Switzerland and let $Y_t = 100 \times \ln\left(GDP_t/GDP_{t-1}\right)$ be the quarterly percentage change in GDP.
Arguably, oil prices adversely affect the economy only when they jump above their values in the recent past: Hence, let O_t the percentage point difference between oil prices at date t and their maximum value during the past year (i.e., $O_t = \max(0, oilpricechange)$).

$$\hat{Y} = \underset{(0.1)}{1.0} - \underset{(0.054)}{0.055}O_t - \underset{(0.057)}{0.026}O_{t-1} - \underset{(0.048)}{0.031}O_{t-2} - \underset{(0.042)}{0.109}O_{t-3} - \underset{(0.053)}{0.128}O_{t-4}$$
$$+ \underset{(0.025)}{0.038}O_{t-5} + \underset{(0.048)}{0.025}O_{t-6} - \underset{(0.039)}{0.019}O_{t-7} + \underset{(0.042)}{0.067}O_{t-8}$$

(a) Suppose that oil prices jump 25% above their previous peak value and stay at this new higher level (so that $O_t = 25$ and $O_{t+1} = O_{t+2} = .. = 0$). What is the predicted effect on output growth for each quarter over the next 2 years?

See the table below. With the 25% oil price jump, the predicted effect on output growth for ith quarter is $25\hat{\beta}_i$ percentage points.

Period ahead (i)	Dynamic multiplier ($\hat{\beta}_i$)	Predicted effect on output growth ($25\hat{\beta}_i$)	95% confidence interval $25[\hat{\beta}_i \pm 1.96SE(\hat{\beta}_i)]$
0	-0.055	-1.375	[-4.033 , 1.283]
1	-0.026	-0.65	[-3.456 , 2.156]
2	-0.031	-0.775	[-3.138 , 1.588]
3	-0.109	-2.725	[-4.792 , -0.66]
4	-0.128	-3.2	[-5.809 , -0.59]
5	0.038	0.95	[-0.281 , 2.181]
6	0.025	0.625	[-1.738 , 2.988]
7	-0.019	-0.475	[-2.395 , 1.445]
8	0.067	1.675	[-0.392 , 3.742]

(b) What intuition can you suggest for the researcher stopping at 8 quarters?

The impact of oil price changes on GDP growth can only be predicted for 2 years. because other factors can also affect GDP growth. Therefore, in general, modeling is performed only for about 8 quarters.

(c) Construct a 95% confidence interval for your answers in (a), assuming we have data from 1955 to 2020.

$n = 66 \times 4 = 264$
$k = 10$
$df = n - k = 264 - 10 = 254$
$\alpha = 0.05$
$t_{(\alpha,df)} = t_{(0.05,264)} = 1.96$

The 95% confidence interval for the predicted effect on output groth for the ith quarter from the 25% oil price jump is $25[\hat{\beta}_i \pm 1.96SE(\hat{\beta}_i)]$ percentage points. The confidence interval is reported in the table in (a).

(d) What is the predicted cumulative change in GDP growth over eight quarters?

The predicted cumulative change in GDP growth over 8 quarters is

$$25 \times (-0.055 - 0.026 - 0.031 + \cdots + 0.067) = 25 \times (-0.238) = -5.95$$

This means that GDP growth has decreased by 5.95% over a two-year period.

(e) A robust F-statistic, used to test whether the coefficients on O_t and its lags are zero, is 3.49. Are the coefficients significantly different from zero? Explain your answer. Briefly indicate why a robust F-statistic was used instead of the usual F statistic.

$$\begin{cases} H_0 : \hat{\beta}_0 = \hat{\beta}_1 = \cdots = \hat{\beta}_8 = 0 \\ H_1 : \hat{\beta}_0 \neq 0 \text{ or } \hat{\beta}_1 \neq 0 \text{ or } \cdots \text{ or } \hat{\beta}_8 \neq 0 \end{cases}$$

$$F_{(k-1,n-k)} = F_{(8,255)} = 1.9 < 3.49$$

where $n = 264$, $k = 9$, A robust F-statistic $= 3.49$
Hence, reject H_0. The coefficients are significantly different from zero.
Robust F-statistic is required because the omitted variable is included in the error term, so that the error term has heteroskedasticity. Also error term is serially correlated(Auto-correlated Errors).

Question 2. Researcher A would like to build a model to estimate elasticity of substitution:

$$\log(\text{ Demand of Sprits }) = \beta_1 + \beta_2 \log(\text{ Price of Beer}) + \beta_3 \log(\text{Price of Spirits}) + u_1$$

added its own price for the control of income effect. Researcher B argues that the demand quantity has to be scaled by price, because no scaling may create larger error.

$$\log \frac{(\text{Demand of Spirits})}{(\text{Price of Spirits})} = \alpha_1 + \alpha_2 \log(\text{Price of Beer}) + \alpha_3 \log(\text{Price of Spirits}) + u_2$$

where u_1 and u_2 are disturbance terms.

(a) Evaluate the researcher A's claim
In order to estimate the elasticity of substitution, (Demand of Beer) variable which is necessary to separate the substitution effect from the income effect should also be added.

(b) Evaluate the researcher B's claim

$$\begin{aligned} \log \frac{(\text{Demand of Spirits})}{(\text{Price of Spirits})} &= \log\ (\text{Demand of Spirits})\ - \log(\text{ Price of Spirits}) \\ &= \alpha_1 + \alpha_2 \log(\text{Price of Beer})\ + \alpha_3 \log\ (\text{Price of Spirits})\ + u_2 \end{aligned}$$

$$\therefore \log(\text{Demand of Spirits}) = \alpha_1 + \alpha_2 \log(\text{Price of Beer})\ + (\alpha_3 + 1) \log(\text{Price of Spirits})\ + u_2$$

Researcher B's model is almost identical to that of Researcher A.(except for only $\alpha_3 + 1$)
Also, since the logarithmic form performs scaling, it is meaningless to divide the variable in the log by Price of Spirits.

(c) How do you interpret $\beta_2, \beta_3, \alpha_2$, and α_3 ? Would B's R^2 be any better than A's?

$$\begin{cases} \alpha_2 = \beta_2 \\ \alpha_3 + 1 = \beta_3 \end{cases}$$

α_2 and β_2 are the price elasticity of Demand of Beer.
α_3 and β_3 are the price elasticity of Demand of Spirits.

β_2 means the effect of Price of Beer on Demand of Spirits,
β_3 means the effect of Price of Spirits on demand of Spirits.
α_2 means the effect of Price of Beer on Demand of Spirits per Price of Spirits,
α_3 means the effect of Price of Spirits on demand of Spirits per Price of Spirits.

$TSS = RSS + ESS$
RSS is same.($u_1 = u_2$)
TSS is different because the dependent variable is different.
Hence, R^2 is different and we don't know which one's R^2 is better.

(d) Researcher C proposes that demand quantity of beer must be included in the regression in order to avoid mis-specification error. Evaluate the claim

C's claim is right. The substitution effect of A's model is mixed with the income effect in the β_2 and β_3. By adding the Demand of Beer to the regression, the income effect and the substitution effect can be separated.

(e) Researcher D claims that there has been a sweeping trend in the country for cocktail that created a variety of mixture of beer and spirit, which has to be reflected in the regression. If you agree with her, how would you include the factor into the model and what are the needed data? Does the form of data pre-processed, if you continue to use logged form of regression?

Suppose

$$I = \begin{cases} 1 & \text{(cocktail)} \\ 0 & \text{(otherwise)} \end{cases}$$

Then we have

$$\begin{aligned} \log(\text{Demand of Spirits}) = \beta_1 &+ \beta_2 \log(\text{ Price of Beer }) + \beta_3 \log(\text{Price of Spirits}) \\ &+ \beta_4 \log\ (\text{Demand of Beer})\ + \beta_5 \cdot I \cdot \log(\text{Demand of Beer}) + u_1 \end{aligned}$$

We need data to know that people drank cocktails. However, it will be difficult in reality. For example, if there is data on soju and beer orders, it is impossible to determine whether people just drank them separately or mixed them.