

Question 7

3.

$$Y_{it} = \hat{\beta}_0 + \hat{\beta}_1 T_i + \hat{\beta}_2 A_t + \hat{\beta}_3 (A_t \times T_i) + \hat{\epsilon}_{it}$$

When $(i, t) = (0, 1)$, $T_i = 0$, $A_t = 0$, $T_i \times A_t = 0$

$Y_{01} = \hat{\beta}_0 + \hat{\epsilon}_{01}$: control group at t=1

When $(i, t) = (0, 2)$, $T_i = 0$, $A_t = 1$, $T_i \times A_t = 0$

$Y_{02} = \hat{\beta}_0 + \hat{\beta}_2 + \hat{\epsilon}_{02}$: control group at t=2, including time effect

When $(i, t) = (1, 1)$, $T_i = 1$, $A_t = 0$, $T_i \times A_t = 0$

$Y_{11} = \hat{\beta}_0 + \hat{\beta}_1 + \hat{\epsilon}_{11}$: treatment group at t=1

When $(i, t) = (1, 2)$, $T_i = 1$, $A_t = 1$, $T_i \times A_t = 1$

$Y_{12} = \hat{\beta}_0 + \hat{\beta}_1 + \hat{\beta}_2 + \hat{\beta}_3 + \hat{\epsilon}_{12}$: treatment group at t=2, including time effect and treatment effect

If there's no selection bias, $Y_{01} = Y_{11}$. Therefore, $\hat{\beta}_1$ should be 0

$\hat{\beta}_2$: time effect

$\hat{\beta}_3$: treatment effect

4.

$$Y_{it} = \hat{\beta}_0 + \hat{\beta}_1 T_i + \hat{\beta}_2 A_{2t} + \hat{\beta}_3 (A_{2t} \times T_i) + \hat{\beta}_4 A_{1t} + \hat{\beta}_5 (A_{1t} \times T_i) + \hat{\epsilon}_{it}$$

If there are 3 periods, $t = 0, 1, 2$

$(i, t) = (0, 0)$: control group at t=0

$Y_{00} = \hat{\beta}_0 + \hat{\epsilon}_{00}$

$(i, t) = (0, 1)$: control group at t=1, including time effect between t=0 and t=1

$Y_{01} = \hat{\beta}_0 + \hat{\beta}_4 + \hat{\epsilon}_{01}$

$(i, t) = (0, 2)$: control group at t=2, including time effect between t=0 and t=2

$Y_{02} = \hat{\beta}_0 + \hat{\beta}_2 + \hat{\epsilon}_{02}$

$(i, t) = (1, 0)$: treatment group at t=0

$Y_{10} = \hat{\beta}_0 + \hat{\beta}_1 + \hat{\epsilon}_{10}$

$(i, t) = (1, 1)$: treatment group at t=1, including time effect between t=0 and t=1 (no treatment effect)

$Y_{11} = \hat{\beta}_0 + \hat{\beta}_1 + \hat{\beta}_4 + \hat{\beta}_5 + \hat{\epsilon}_{11}$

$(i, t) = (1, 2)$: treatment group at $t=2$, including time effect between $t=0$ and $t=2$ and treatment effect

$$Y_{12} = \hat{\beta}_0 + \hat{\beta}_1 + \hat{\beta}_2 + \hat{\beta}_3 + \hat{\epsilon}_{12}$$

$\hat{\beta}_1$: Treatment group's effect (If there's no selection bias, should be 0)

$\hat{\beta}_2$: Time effect between $t=0$ and $t=2$

$\hat{\beta}_3$: Treatment effect

$\hat{\beta}_4$: Time effect between $t=0$ and $t=1$

$\hat{\beta}_5$: Treatment group's effect at $t=1$, which should be 0

(Assuming no selection bias, $Y_{01} = Y_{11}$. The treatment occurs between time $t = 1$ and $t = 2$)

Hence, if $\hat{\beta}_5 = 0$ is true, we can say there's no selection bias, which satisfies assumptions from (3)