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Question 1. Let $gGDP_t$ denote the annual percentage change in gross domestic product and let int_t denote a short-term interest rate. Suppose that $gGDP_t$ is related to interest rate by

$$gGDP_t = \alpha_0 + \delta_0 int_t + \delta_1 int_{t-1} + u_t$$

where u_t is uncorrelated with int_t, int_{t-1}, and all other past values of interest rates.

Suppose that the Federal Reserve follows the policy rule:

$$int_t = \gamma_0 + \gamma_1(gGDP_{t-1} - 3) + v_t$$

where $\gamma_1 > 0$. In behavioral terms, $\gamma_1 > 0$ means that when last year's GDP growth is above 3%, the Fed increases interest rates to prevent an "overheated" economy.

(a) If v_t is uncorrelated with all past values of int_t and u_t, argue that int_t must be correlated with u_{t-1}. (Hint: Lag the first equation for one time period and substitute for $gGDP_{t-1}$ in the second equation.) We want to show that $Cov(int_t, u_{t-1}) \neq 0$

[Answer]
u_t is uncorrelated with int_t, int_{t-1}, and all other past values of interest rates. ($u_t \perp int_t, int_{t-1}, int_{t-2}$)

$$\begin{aligned} Corr(u_t, int_t) &= 0 \\ Corr(u_t, int_{t-1}) &= 0 \\ Corr(u_t, int_{t-2}) &= 0 \end{aligned}$$

If lag u_t for one time period

$$\begin{aligned} Corr(u_{t-1}, int_{t-1}) &= 0 \\ Corr(u_{t-1}, int_{t-2}) &= 0 \end{aligned}$$

lag the first equation for one time period,

$$\begin{aligned} gGDP_t &= \alpha_0 + \delta_0 int_t + \delta_1 int_{t-1} + u_t \\ gGDP_{t-1} &= \alpha_0 + \delta_0 int_{t-1} + \delta_1 int_{t-2} + u_{t-1} \qquad (1) \end{aligned}$$

substitute for $gGDP_{t-1}$ in the second equation,

$$\begin{aligned} int_t &= \gamma_0 + \gamma_1(gGDP_{t-1} - 3) + v_t \\ &= \gamma_0 + \gamma_1(\alpha_0 + \delta_0 int_{t-1} + \delta_1 int_{t-2} + u_{t-1} - 3) + v_t \\ &= (\gamma_0 + \gamma_1\alpha_0 - 3\gamma_1) + \gamma_1\delta_0 int_{t-1} + \gamma_1\delta_1 int_{t-2} + (\gamma_1 u_{t-1} + v_t) \quad \cdots \text{ AR(2) model} \qquad (2) \end{aligned}$$

Here, int_t and u_{t-1} are in the same row.

$$Corr(int_t, u_{t-1}) \neq 0$$

Hence

$$Cov(int_t, u_{t-1}) \neq 0$$

(b) Which Gauss-Markov assumption does $Corr(int_t, u_{t-1})$ violate?
$Corr(int_t, u_{t-1}) \neq 0$ violates A3Rmi that $E[\epsilon \mid X] = 0 \;\; \forall int_t, u_t$
And

$$\begin{aligned} Corr(u_{t-1}, int_{t-1}) &= 0 \\ Corr(u_{t-1}, int_{t-2}) &= 0 \end{aligned}$$

This means A3Rsru (same row uncorrelated) does not violated.
Therefore, if the sample size is small, the estimator is biased, but if the sample size is large, the estimator is consistent.