

1. (a) For  $\beta_3$  to be consistently estimated,  $A3R_{su}$  should be satisfied in the first equation. This means that  $\text{Cov}(\text{marijuana}, u_1) = 0$   $\text{Cov}(\text{educ}, u_1) = 0$  should be satisfied.

However, it isn't due to simultaneity between marijuana and earnings. Therefore, we use fine, prison as IVs.

IVs should satisfy validity and relevance.

① Validity:  $\text{Cov}(\text{fine}, u_1) = \text{Cov}(\text{prison}, u_1) = 0$

② Relevance:  $\text{Cov}(\text{fine}, \text{marijuana}) \neq 0$ ,  $\text{Cov}(\text{prison}, \text{marijuana}) \neq 0$

The higher the relevance is, the higher the precision of  $\hat{\beta}_3$ s are.

(b) We can use 2SLS to estimate  $\beta_3$ .

① First stage:  $\text{marijuana} = \pi_{m1} \text{fine} + \pi_{m2} \text{prison} + \delta \text{educ} + v$

From the first stage, we can obtain  $\widehat{\text{marijuana}} = \hat{\pi}_{m1} \text{fine} + \hat{\pi}_{m2} \text{prison} + \hat{\delta} \text{educ}$ .

② Second stage:  $\log(\text{earnings}) = \beta_0 + \beta_1 \widehat{\text{marijuana}} + \beta_2 \text{educ} + u_1$

From the second stage, we can obtain  $\hat{\beta}_{0,2SLS}$ ,  $\hat{\beta}_{1,2SLS}$ ,  $\hat{\beta}_{2,2SLS}$  consistently since  $\widehat{\text{marijuana}}$  and  $\text{educ}$  are uncorrelated with  $u_1$  and  $A3R_{su}$  is satisfied.

(c) Since we have 1 endogenous variable, marijuana, and 2 IV candidates, fine & prison, we have overidentification

2. (a) The coefficient of  $\log(\text{avgprc}_t)$  means the price elasticity of demand, which is  $\frac{\% \text{ change in totqty}}{\% \text{ change in avgprc}}$ . It focuses on the ratio of change "rate" on average.

Since t-statistic of the coefficient is about -2.5 and its magnitude is larger than 2, we can think  $\log(\text{avgprc})$  is significant. In addition, the negative sign of the coefficient corresponds with our intuition that quantity for demand decreases as average price increases.

(b) Simultaneity is one of a kind of endogeneity and breaks A3R condition. This can lead to biasedness & inconsistency of coefficients. In this case,  $\log(\text{avgprc}_t)$  is an endogenous variable. System of equations are described as below.

$$\begin{cases} \log(\text{totqty}_t) = \alpha_1 + \alpha_2 \log(\text{avgprc}_t) + \alpha_3 \text{mon}_t + \alpha_4 \text{tues}_t + \alpha_5 \text{wed}_t + \alpha_6 \text{thurs}_t + U_{dt} \\ \log(\text{totqty}_t) = \beta_1 + \beta_2 \log(\text{avgprc}_t) + U_{st} \end{cases} \begin{array}{l} \text{demand} \\ \text{Supply} \end{array}$$

Since the demand equation does not satisfy order condition, we cannot identify it now. We should introduce exogenous variables included only in Supply equation.

(c) To use  $\text{wave2}_t$  &  $\text{wave3}_t$  as IVs, those variables should satisfy validity and relevance

$$\textcircled{1} \text{ Validity: } \text{Cov}(\text{wave2}_t, U_{st}) = \text{Cov}(\text{wave2}_t, U_{dt}) = \text{Cov}(\text{wave3}_t, U_{st}) = \text{Cov}(\text{wave3}_t, U_{dt}) = 0$$

$$\textcircled{2} \text{ Relevance: } \begin{cases} \text{Cov}(\text{wave2}_t, \log(\text{avgprc}_t)) \neq 0 \\ \text{Cov}(\text{wave3}_t, \log(\text{avgprc}_t)) \neq 0 \end{cases}$$

$\text{wave2}_t$  and  $\text{wave3}_t$  should not be in demand equation

Wave heights would affect only the quantity for supply, not the quantity for demand because large wave heights for past two or three days make fishermen not be able to go to sea, decreasing the quantity for supply and increasing the daily price.

Therefore, the assumptions seem reasonable.

The structural equations are changed as below.

$$\begin{cases} \log(\text{totqty}_t) = \alpha_1 + \alpha_2 \log(\text{avgprc}_t) + \alpha_3 \text{mon}_t + \alpha_4 \text{tues}_t + \alpha_5 \text{wed}_t + \alpha_6 \text{thurs}_t + u_{dt} \\ \log(\text{totqty}_t) = \beta_1 + \beta_2 \log(\text{avgprc}_t) + \beta_3 \text{wave2}_t + \beta_4 \text{wave3}_t + u_{st} \end{cases}$$

- demand  
- Supply

To obtain  $\alpha$ s consistently, we would use 2SLS method

① First stage

$$\begin{aligned} \log(\text{avgprc}_t) &= \pi_0 + \pi_1 \text{wave2}_t + \pi_2 \text{wave3}_t + \pi_3 \text{mon}_t + \pi_4 \text{tues}_t + \pi_5 \text{wed}_t + \pi_6 \text{thurs}_t \\ \rightarrow \widehat{\log(\text{avgprc}_t)} &= \hat{\pi}_0 + \hat{\pi}_1 \text{wave2}_t + \hat{\pi}_2 \text{wave3}_t + \hat{\pi}_3 \text{mon}_t + \hat{\pi}_4 \text{tues}_t + \hat{\pi}_5 \text{wed}_t + \hat{\pi}_6 \text{thurs}_t + e_t \end{aligned}$$

② Second stage

$$\log(\text{totqty}_t) = \alpha_1 + \alpha_2 \widehat{\log(\text{avgprc}_t)} + \alpha_3 \text{mon}_t + \alpha_4 \text{tues}_t + \alpha_5 \text{wed}_t + \alpha_6 \text{thurs}_t + u_{dt}^*$$

$$(d) \text{ t-statistic of wave2}_t = -\frac{1.022}{0.144} < -2$$

$$\text{t-statistic of wave3}_t = -\frac{1.022}{0.144} < -2$$

$$F\text{-statistic} = \frac{(RSS - URSS)/df}{URSS/(n-p)} = \frac{(15.576 - 10.934)/2}{10.934/(91-1)} = 19.1 \sim F(2, 90)$$

$$H_0: \pi_1 = \pi_2 = 0 \quad H_A: \text{not } H_0$$

Therefore, we can reject  $H_0$  and conclude that wave2<sub>t</sub> & wave3<sub>t</sub> are significant both in t-tests & F-test.

This result corresponds to our assumptions that wave2<sub>t</sub> & wave3<sub>t</sub> have relevance with  $\log(\text{argprc}_t)$

(e) As I mentioned in (c), 2SLS method is implemented as below.

① First stage

$$\begin{aligned} \log(\text{argprc}_t) &= \pi_0 + \pi_1 \text{wave2}_t + \pi_2 \text{wave3}_t + \pi_3 \text{mon}_t + \pi_4 \text{tues}_t + \pi_5 \text{wed}_t \\ &\rightarrow \widehat{\log(\text{argprc}_t)} = \hat{\pi}_0 + \hat{\pi}_1 \text{wave2}_t + \hat{\pi}_2 \text{wave3}_t + \hat{\pi}_3 \text{mon}_t + \hat{\pi}_4 \text{tues}_t + \hat{\pi}_5 \text{wed}_t + \hat{\pi}_6 \text{thurs}_t + e_t \end{aligned}$$

② Second stage

$$\log(\text{totqty}_t) = \alpha_1 + \alpha_2 \widehat{\log(\text{argprc}_t)} + \alpha_3 \text{mon}_t + \alpha_4 \text{tues}_t + \alpha_5 \text{wed}_t + \alpha_6 \text{thurs}_t + u_{dt}^*$$

As a result,  $\alpha_j$ s ( $j=1,2,3,4,5,6$ ) would be same as IV estimates without standard errors. We should use robust standard errors when we use 2SLS method.