TA session 4

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1 QnA

Question 1. In AR(1) model, A3Rmi assumption isn't satisfied because y_{t-1} includes ϵ_{t-1}?

Question 2. In the model $y_t = \alpha_0 + \beta_1 z_t + \beta_2 z_{t-1} + u_t$, A3Rmi assumption isn't satisfied because there is multicollinearity?

2 Problem Set

2.1 What do the problems mean in the assignment?

Q1. If there are lag variables in the model, we should check whether the relation btwn lag variables and error. (Simultaneity)

Q2. (b)

$$\begin{aligned}
E(\hat{\beta}) &= E((y'_{t-1}y_{t-1})^{-1}y'_{t-1}y_t) = E((y'_{t-1}y_{t-1})^{-1}y'_{t-1}(\beta_1 y_{t-1} + \epsilon_t)) \\
&= E((y'_{t-1}y_{t-1})^{-1}y'_{t-1}\beta_1 y_{t-1}) + E((y'_{t-1}y_{t-1})^{-1}y'_{t-1}\epsilon_t) \\
&= \beta_1 + E(E((y'_{t-1}y_{t-1})^{-1}y'_{t-1}\epsilon_t|y_1, y_2, ..., y_{t-1}, y_t, ..., y_T)) \quad \because \text{Law of Iterated Expectation} \\
&= \beta_1 + E((y'_{t-1}y_{t-1})^{-1}y'_{t-1}E(\epsilon_t|y_1, y_2, ..., y_{t-1}, y_t, ..., y_T)) \\
&= \beta_1 + c, \ \ c : \text{constant} \ \ \because E(\epsilon_t|y_t, y_{t+1}, ...) \neq 0 \\
&\neq \beta_1
\end{aligned}$$

(c)

$$\begin{aligned}
\text{plim}\,\hat{\beta} &= \text{plim}(y'_{t-1}y_{t-1})^{-1}y'_{t-1}y_t \\
&= \text{plim}(y'_{t-1}y_{t-1})^{-1}y'_{t-1}(\beta_1 y_{t-1} + \epsilon_t) \\
&= \beta_1 + \text{plim}(y'_{t-1}y_{t-1})^{-1}y'_{t-1}\epsilon_t \\
&= \beta_1 + \text{plim}\left(\frac{y'_{t-1}y_{t-1}}{N}\right)^{-1}\left(\frac{y'_{t-1}\epsilon_t}{N}\right) \\
&= \beta_1 + \text{plim}\left(\frac{y'_{t-1}y_{t-1}}{N}\right)^{-1}\text{plim}\left(\frac{y'_{t-1}\epsilon_t}{N}\right) \\
&= \beta_1 + (Var(y_{t-1}))^{-1}(Cov(y_{t-1}, \epsilon_t)) \\
&= \beta_1 + (Var(y_{t-1}))^{-1}(E(y'_{t-1}\epsilon_t)) \quad \because Cov(y_{t-1}, \epsilon_t) = E(y'_{t-1}\epsilon_t) - E(y'_{t-1})E(\epsilon_t) = E(y'_{t-1}\epsilon_t) \\
&= \beta_1 \quad \because \epsilon_t \text{ is independent } y_{t-1} \\
&: \text{ A3Rsru holds. So we need large sample to get true parameter!}
\end{aligned}$$

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3 Recap

1. Differences between cross-sectional and time series data

   a. Does time passes?

   b. Seasonal dummy variable (including intercations): **Seasonally adjusted**
   ex. $y_t = \alpha_0 + \beta x_t + \gamma_1 s_{1t} + \gamma_2 s_{2t} + \gamma_3 s_{3t} + \epsilon_t$

   c. Multicollinearity problem by dummies

   d. Supurious Regression problem with common tendency or effect

2. Finite Distributed Lag (FDL) Models
   : FDL model is good for estimtating lagged effects of variable(including dependent). Especially the effect is unlikely instantaneous. ex. $y_t = \alpha_0 + \delta_0 z_t + \delta_1 z_{t-1} + \delta_2 z_{t-2} + u_t$

   – FDLs are often more realistic than static models because they account for some dynamic behavior.

3. Models with Lagged Dependent Variables (ARMA(p,q)):

   a. Autoregressive model: past outcomes on y affect current y. There are short-term effect and long-term effect

   b. OLS estimators are no longer unbiased(finite sample property). So use large-sample analysis(infinite sample property, consistent, plim)

   c. Moving Average: a succession of averages derived from successive segments (typically of constant size and overlapping) of a series of values.

3. Finite-Sample Analysis of OLS for TS Data: Gauss Markov Assumptions for Time-Series

   (A1) No Perfect Collinearity: rules out perfect linear relations among the explanatory variables. Near multicollinearity can yield unreliable parameter

   (A2) Linear in Parameters: Linear relation between explanatory variable and dependent variable $E(\epsilon) = 0$

   (A3) Relation between explanatory variables and error

   * Zero conditional Mean: $\mathbb{E}(u_t|X) = \mathbb{E}(u_t|x_1, x_2, ..., x_t, ..., x_T)$, x_t : row vector for each t
     u_t is uncorrelated with each x_{sj} for all t and s, including $t = s$ and all varables j is the index of columns. **(too Strong)**
   * Contemporaneous exogeneity($\mathbb{E}(u_t|X) = 0$)
     : There may be correlation between u_t and x_{t+1}. But it's enough for large-sample properties (consistency)

   (A4)
   * Homoskedasticity: Same Variances
   * No serial Correlation: Correlations between errors should be zero

   (A5) Normality: how does this assumption make exact inference
   How is it possible? $z - dist \rightarrow \chi^2 - dist \rightarrow F - dist$

4. Gauss-Markov Theorem for TS
   : Under (A1), (A2), (A3Rmi), (A4), the OLS estimators are BLUE(the best, linear, unbiased estimators)

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