

**Question 1-(a)**

[Answer] We need the assumption that the linear combination of *educ*, *fine*, and *prison* have relevance to *marijuana*. This is reasonable at least in *fine* and *prison* as people fine or go to prison due to *marijuana*. According to *educ*, it has relevance to  $\log(\text{earning})$  as higher education brings a lot of earning while it is orthogonal to other variables in linear equations. Besides their exogeneity, the linear regression has endogeneity caused by simultaneity from the feedback structure between  $\log(\text{earning})$  and *marijuana*. In order consistently to estimate  $\beta_j$ , we need to suppose that *fine*, and *prison* can remove the simultaneity from the linear regression as instruments.

**Question 1-(b)**

[Answer] First, we transform the second equation:

$$marijuana = \gamma_0 + \gamma_1 \log(\text{earnings}) + \gamma_2 educ + \gamma_3 fine + \gamma_4 prison + u_2 \quad (1)$$

$$\rightarrow \log(\text{earning}) = \frac{\gamma_0}{\gamma_1} - \frac{1}{\gamma_1} marijuana + \frac{\gamma_2}{\gamma_1} educ + \frac{\gamma_3}{\gamma_1} fine + \frac{\gamma_4}{\gamma_1} prison + \frac{1}{\gamma_1} u_2 \quad (2)$$

Then we can have the structural form of simultaneous linear regression equation. Then we run 1st stage linear regression like the following:

$$\widehat{marijuana} = \pi_1 + \pi_2 fine + \pi_3 prison \quad (3)$$

After we estimate  $\widehat{marijuana}$ , we apply  $\widehat{marijuana}$  for removing the simultaneity from *marijuana* in the original linear regression and run OLS on the following equation:

$$\log(\text{earning}) = \beta_0 + \beta_1 \widehat{marijuana} + \beta_2 educ + u_1 \quad (4)$$

Now we can get the proper coefficients  $\beta_j$  after removing simultaneity.

**Question 1-(c)**

[Answer] Yes. For removing the simultaneity from *marijuana*, we use *fine* and *prison*. As we use two factors, we do overidentification.

**Question 2-(a)**

[Answer] Suppose that the coefficient of  $\log(\text{avgprc}_t)$  is  $\beta_2$ . The sign of  $\beta_2$  is minus. This means the negative correlation between  $\log(\text{totqty}_t)$  and  $\log(\text{avgprc}_t)$ . This seems reasonable as the demand quantity is decreased as the fish price goes up or vice versa. The coefficient itself represents the price elasticity on demand as  $\log$  is used on quantity and price. The elasticity consists of two things: substitute effect and income effect. For capturing income effect only, we have to see the substitutes of fish like meat or imported salmon. Even though the variance of  $\beta_2$  is low compared to other coefficients, it is meaningless as the variance of the coefficients in dummy variables is large in general. On  $\alpha = 0.05$ , the t-test on  $H_0: \beta_2 = 0$ ,  $H_1: \beta_2 \neq 0$  can be done and  $H_0$  is rejected as t-stat is significant ( $\frac{-0.425-0}{0.176} = -2.41 < -1.96$ ). However, on  $\alpha = 0.01$ , we cannot reject  $H_0$  as  $-2.41 > -2.58$  and the t-test is insignificant. To see its significance among all variables including  $\log(\text{avgprc}_t)$ , we have to do f-test by comparing the critical value  $\frac{(RRSS-URSS)/1}{URSS/(97-6)}$  with  $F_{1-0.01}(1, 97-6)$ .

**Question 2-(b)**

[Answer] From the quantity and price on supply and demand with simultaneity, we can have the following equations in structural form:

$$\begin{cases} q_i = \alpha_1 p_i + \beta_1 z_i + u_{1i} \\ q_i = \alpha_2 p_i + u_{2i} \end{cases} \quad (5)$$

where  $E(z_i u_{1i}) = 0$ . From its reduced form in equilibrium, we have:

$$\alpha_1 p_i + \beta_1 z_i + u_{1i} = \alpha_2 p_i + u_{2i} \quad (6)$$

Now we have:

$$p_i = \frac{\beta_1}{\alpha_2 - \alpha_1} z_i + \frac{1}{\alpha_2 - \alpha_1} (u_{1i} - u_{2i}) = \pi_p z_i + v_{pi} \quad (7)$$

$$\text{Cov}(p_i, u_{1i}) = \text{Cov}(\pi_p z_i + \frac{1}{\alpha_2 - \alpha_1} (u_{1i} - u_{2i}), u_{1i}) = \frac{\sigma_1^2 - \sigma_{12}}{\alpha_2 - \alpha_1} \neq 0 \quad (8)$$

$$\text{Cov}(p_i, u_{2i}) = \text{Cov}(\pi_p z_i + \frac{1}{\alpha_2 - \alpha_1} (u_{1i} - u_{2i}), u_{2i}) = \frac{\sigma_{12} - \sigma_2^2}{\alpha_2 - \alpha_1} \neq 0 \quad (9)$$

This means the linear regression with simultaneity has endogeneity. Therefore the estimates of the linear regression are wrong and the coefficients are wrong.

**Question 2-(c)**

[Answer] The two assumptions are the followings: First,  $\text{wave2}_t$  and  $\text{wave3}_t$  have no correlation on the error of the linear regression equation (instrumental validity, exogenous explanatory variables). Second,  $\text{wave2}_t$  and  $\text{wave3}_t$  have relevance on  $\log(\text{avgprc}_t)$ . These assumptions are reasonable. The quantity of fish supply depends on the ocean wave height and they can shift the equilibrium points, which reveals the fish demand curve.

**Question 2-(d)**

[Answer] We do f test for checking the significance of two variables  $\text{wave2}$  and  $\text{wave3}$ . F stat is:

$$\frac{(RRSS - URSS)/q}{URSS/(n - p)} = \frac{(15.576 - 10.934)/2}{10.934/(97 - 3)} = 19.915 \quad (10)$$

As  $F_{1-0.05}(2, 94) = 3.09 < 19.915$ , we reject the null hypothesis. The coefficients of the linear regression are not zero. This test result corresponds to the result of 1-(c) as  $\text{wave2}_t$  and  $\text{wave3}_t$  explains  $\log(\text{avgprc}_t)$  well.

**Question 2-(e)**

[Answer] From the first stage regression in 2-(d), we can have  $\widehat{\log(\text{avgprc}_t)}$ . Then we can run regression of the following:

$$\widehat{\log(\text{totqty}_t)} = \beta_1 + \beta_2 \widehat{\log(\text{avgprc}_t)} + \beta_3 \text{mon}_t + \beta_4 \text{tues}_t + \beta_5 \text{wed}_t + \beta_6 \text{thrus}_t \quad (11)$$

From the above regression, we can get  $\beta = (8.164, -0.815, -0.307, -0.685, -0.521, 0.095)'$ .  $\widehat{\log(\text{avgprc}_t)}$  removes the simultaneity part from  $\log \text{avgprc}_t$ , which has the cause of endogeneity. For regressing, we have to use  $\log(\text{avgprc}_t)$ , not  $\widehat{\log \text{avgprc}_t}$  for correct result.